Short Papers

Boundary Control of a Translating Tensioned Beam With Varying Speed

Kyung-Jinn Yang, Keum-Shik Hong, and Fumitoshi Matsuno

***Abstract*—The investigational results for an active vibration control of a translating tensioned beam with a varying traveling speed are presented. The dynamics of beam and actuator is modeled via the extended Hamilton's principle. In a proper mathematical manner, the Lyapunov method is employed to design a boundary control law for ensuring the vibration reduction of the nonlinear time-varying system and also to ensure the exponential stability of the closed-loop system.**

***Index Terms*—Axially moving continua, boundary control, Lyapunov method, stability.**

I. Introduction

The control problem of axially moving continua occurs in such high performance mechanical systems as cranes, strips in a thin-metal-sheet production line, high-rise elevators, chains and belts, high-speed magnetic tapes, and deployable robot arms. However, the unwanted vibrations of moving continua due to the flexibility property and time-varying conditions restrict the utility of the systems in many applications and in particular in high-speed, precision systems. Hence, in this paper, a nonlinear traveling beam with a time-varying speed is particularly focused on, resulting in a problem formulation, an implementable controller design, and a stability analysis.

II. Beam Model: Problem Formulation

Fig. 1 shows the considered axially moving beam with a hydraulic touch-roll actuator at the right boundary. The roll at the left boundary is assumed to be fixed. Let t be the time, x be the spatial coordinate along the longitude of motion, $v(t)$ be the varying axial speed of the beam, $v(t) > 0$ for all t, $w(x,t)$ be the transversal displacement of the beam at spatial coordinate x and time t, and l be the length of the beam. Also, let ρ be the mass per unit length, A be the cross-sectional area, E be the coefficient of elasticity, I be the moment of inertia of the beam cross section, and $T_s(x,t)$ be the spatiotemporally varying tension applied to the beam. Let the mass and damping coefficients of the hydraulic actuator be m_c and d_c, respectively. The control force $f_c(t)$ is applied to the touch rolls to suppress the transverse vibrations of the axially moving beam. The kinetic and potential energies of the axially moving beam between $x = 0$ and $x = l$ are given as, respectively

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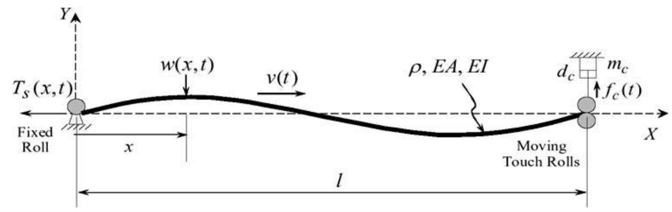

Fig. 1. Schematic of a translating beam subject to a boundary control.

$$T = T_{\text{beam}} + T_{\text{act}} = \frac{\rho A}{2}\int_0^l (w_t + v w_x)^2 dx + \frac{1}{2} m_c w_t^2(l) \quad (1)$$

$$U_{\text{beam}} = \int_0^l \left\{ \frac{T_s(x,t)}{2} w_x^2 + \frac{EA}{8} w_x^4 + \frac{EI}{2} w_{xx}^2 \right\} dx \quad (2)$$

where $w(l) = w(l,t)$ for notational brevity.

The equations of motion and their respective boundary conditions can be obtained through Hamilton's principle. However, in translating systems, the configurations at the end times of the variational principle are not prescribed [1]. Hence, new approaches for d'Alembert's principle are required and can be accomplished by introducing a general theory for calculating the time rate of change [2]. That is, by employing the general theory to the variational principle, the property in the system volume is converted in terms of that in the control volume. Because the configurations in the control volume are prescribed at the specific times in Eulerian description, a novel extended Hamilton's principle for the translating continua systems can be established without loss of generality of the classic Hamilton's principle, which is given as [3]

$$\delta \int_{t_0}^{t_f} L dt + \int_{t_0}^{t_f} \delta W_{n.c.} dt - \int_{t_0}^{t_f} \rho A (w_t(l) + v w_x(l)) \cdot v \delta w(l)\, dt = 0 \quad (3)$$

where $L = T - U_{\text{beam}}$ and $\delta W_{n.c.} = f_c \delta w(l) - d_c w_t(l) \delta w(l)$.

From (3), the equations of motion and boundary conditions of the axially moving beam system in Fig. 1 are derived as

$$\rho A (w_{tt} + \dot{v} w_x + 2 v w_{xt} + v^2 w_{xx}) - \left((T_s w_x)_x + \frac{3EA}{2} w_x^2 w_{xx} \right) + EI w_{xxxx} = 0 \quad (4)$$

$$w(0) = 0, \quad w_x(0) = 0, \quad w_{xx}(l) = 0 \quad (5)$$

$$m_c w_{tt}(l) + d_c w_t(l) + T_s(l) w_x(l) + \frac{EA}{2} w_x^3(l) - EI w_{xxx}(l) = f_c(t). \quad (6)$$

Note that (4) is a nonlinear hyperbolic partial differential equation representing the transverse motion and (6) is an ordinary differential equation describing the motion of the hydraulic actuator in compliance with the transversal control force at $x = l$. The term $(T_s + 3EAw_x^2/2)$ in (4) is often called a nonlinear tension. The moving speed v, to avoid a divergence of the solution, should be smaller than the critical speed [3]. The tension $T_s(x,t)$ in (4) is given as

$$T_s(x,t) = T_0 - \rho A (l - x)(eg - \dot{v}) \quad (7)$$

where $e = 0$ for the horizontally translating beam, $e = 1$ for the vertically translating beam, and g and T_0 denote the gravitational acceleration and the initial tension applied to the beam, respectively. Note that the axial force may become a tensile and compressive force during the deceleration ($\dot{v} < 0$) and acceleration ($\dot{v} > 0$), respectively, of the beam.

Since the tension $T_s(x,t)$ is a spatiotemporally varying function, the tension variation has to be incorporated into the control law design. Provided that there is no big disturbance in the system, $T_s(x,t)$ can be assumed to be continuous and uniformly bounded, $0 < T_{s,\min} \le T_s(x,t) \le T_{s,\max}$, $|(T_s)_t| \le (T_s)_{t,\max}$, and $|(T_s)_x| \le (T_s)_{x,\max}$ for all $x \in [0,l], t \ge 0$, and some *a priori* known constants $T_{s,\min}, T_{s,\max}, (T_s)_{t,\max}$, and $(T_s)_{x,\max}$, where $(T_s)_t = \rho A(l-x)\ddot{v}$ and $(T_s)_x = \rho A(eg - \dot{v})$ from (7). Considering practical situations such as a high-tensioned beam under axial transport processing, it can be assumed that the lower bound $T_{s,\min}$ is larger than both $(T_s)_{t,\max}$ and $(T_s)_{x,\max}$ due to the high tension limit [4].

Now, consider the open-loop controlled beam system in (4)–(6) with the assumption of $f_c = 0$. From (1) and (2), the total vibration energy $E(t)$ of the beam system is given by

$$E(t) = (T_{\text{beam}} + U_{\text{beam}}) + T_{\text{act}} = E_{\text{beam}} + T_{\text{act}}. \tag{8}$$

Employing the time derivation method in [2] to $E_{\text{beam}}(t)$ in (8) yields

$$\dot{E}_{\text{beam}}(t) = \int_0^l \frac{\partial}{\partial t}\tilde{E}_{\text{beam}}(x,t)dx + v\tilde{E}_{\text{beam}}(x,t)\,|_0^l \tag{9}$$

where

$$\tilde{E}_{\text{beam}}(x,t) = \left[\frac{\rho A}{2}(w_t + vw_x)^2 + \frac{T_s}{2}w_x^2 + \frac{EA}{8}w_x^4 + \frac{EI}{2}w_{xx}^2\right].$$

Hence, the time derivation of $E(t)$ in (8) is evaluated as

$$\begin{aligned}\dot{E}(t) &= -EIvw_{xx}^2(0) - d_c w_t^2(l) \\ &\quad + vw_x^2(l)\left\{T_s(l) + \frac{EA}{2}w_x^2(l)\right\} \\ &\quad - vw_x(l)EIw_{xxx}(l) + \frac{1}{2}\int_0^l \{(T_s)_t + v(T_s)_x\}w_x^2 dx.\end{aligned} \tag{10}$$

From (10), it is justified that the time rate of change of $T_s(x,t)$ should be properly handled to decrease the vibration energy, and it is also seen that the stability of an open-loop system controlled only by a damper cannot be clearly decided, except for the stationary continua system, i.e., $v = 0$.

III. Control Law

As shown in (4)–(6), the control mechanism is coupled with the beam system because the controller is attached to the boundary of the beam, on which the control force f_c is applied. To obtain the stability of coupled system (4)–(6), a modification of the total mechanical energy is necessary to have an appropriate Lyapunov function candidate for the coupled system.

The beam vibration energy E_{beam} in (8) and the following function are equivalent [4]:

$$V_{\text{beam}} = E_{\text{beam}} + \beta\rho A \int_0^l xw_x(w_t + vw_x)\,dx \tag{11}$$

where $0 < \beta < \min\{l^{-1}v, \beta_1^{-1}\}$ and $\beta_1 = l \cdot \max\{1, \rho A T_{s,\min}^{-1}\}$. Note that, according to (5) and Poincare's inequality, the stability of the hydraulic actuator system can be also analyzed by adding the slope term at $x = l$ to the mechanical energy. Thus, a positive definite functional $V(t)$, as the total energy of the moving beam system including the actuator, is defined as

$$V(t) = V_{\text{beam}} + V_{\text{act}} \tag{12}$$

where $V_{\text{act}} = m_c\{w_t(l) + (v + \beta l)w_x(l)\}^2/2$. In this paper, the functional $V(t)$ in (12) is considered as a Lyapunov function candidate to ensure that the desired final state, $(w, \dot{w}, w(l), w_t(l)) = (0,0,0,0)\,|_{\text{desired}}$, is the unique minimum of $V(t)$ in (12).

Now, the control law for the right boundary control force $f_c(t)$ is proposed as

$$\begin{aligned}f_c(t) &= -m_c\{\dot{v}w_x(l) + (v + \beta l)w_{xt}(l)\} \\ &\quad + d_c w_t(l) - \frac{\beta\rho A l v}{v + \beta l}w_t(l).\end{aligned} \tag{13}$$

By employing the derivation method in (9) to evaluate the time rate of change of $V(t)$ in (12), the main theorem of this paper is established.

Theorem 1: Suppose $\{\beta(T_{s,\min} - \rho A v^2) - (T_s)_{t,\max} - (\beta l + v)(T_s)_{x,\max}\} > 0$ and $T_s(l) - \rho A v^2 > 0$. Then, the dynamics of the closed-loop system with the control input $f_c(t)$ in (13) is exponentially stable, i.e.,

$$V(t) \le V(0)e^{-\lambda t} \tag{14}$$

where $\lambda > 0$ and

$$\lambda = \min\begin{bmatrix} \frac{3\beta}{(1+\beta\beta_1)}, \frac{\beta\left(T_{s,\min} - \rho A v^2\right) - (T_s)_{t,\max} - (\beta l + v)(T_s)_{x,\max}}{2T_{s,\max}(1+\beta\beta_1)} \\ \frac{\beta}{2(1+\beta\beta_1)}, \frac{\beta(T_{s,\min} - \rho A v^2) - (T_s)_{t,\max} - (\beta l + v)(T_s)_{x,\max}}{4\rho A v^2(1+\beta\beta_1)} \\ \frac{\beta l(T_s(l) - \rho A v^2)}{2m_c(v+\beta l)^2}, \frac{\beta\rho A l}{m_c}\left(\frac{v}{v+\beta l} - \frac{1}{2}\right)\end{bmatrix}.$$

Note that the dynamic model of a translating string with an arbitrarily varying speed can be easily obtained by setting $E = 0$ in the beam model (4)–(6). Hence, the proposed boundary controller can be directly applied to the axially moving string system without any modifications for ensuring the vibration reduction.

Boundary control law (13) is given for velocity $w_t(l)$, slope $w_x(l)$, and slope rate $w_{xt}(l)$ at $x = l$. By using an encoder (or photodiode) on the actuator and two laser sensors, the actuator displacement $w(l)$ and the slope $w_x(l)$ on the actuator, respectively, can be measured (see [5], [6]). The actuator velocity $w_t(l)$ and the slope rate $w_{xt}(l)$ can then be implemented by the backward differencing of the signals.

IV. Simulations and Discussion

The effectiveness of the proposed control law and the verification of the introduced theories are demonstrated by numerical simulations. For numerical simulations, consider the dimensionless variables (see [3]), and then the parameters of the beam and actuator in (4)–(6) are given as $\rho A = 1, l = 1, m_c = 0.5, T_o = 10, EI = 1, EA = 1$, and $e = 0$. Let the initial conditions of the beam satisfying the boundary conditions in (5) be $w(x,0) = 10^2 \cdot x^2 \cdot (0.5 \cdot l - x)^3 \cdot (l - x)^2$ and $w_t(x,0) = 0$.

A. *Effect of Varying Speed $v(t)$ in Uncontrolled Systems*

Fig. 2 depicts three type energies of the dimensionless beam without both control force and damper, i.e., $f_c = d_c = 0$ in (6) under $v = 1$ (solid line), $v(t) = 1 + 0.5\sin 10t$ (dashed line), and $v(t) = 1 + 0.5\sin 35t$ (dotted line), respectively. As shown in Fig. 2, the difference of vibration energy between $v = 1$ and $v(t) = 1 + 0.5\sin 10t$ is not so big despite the varying condition. However, the magnitude of the energy under $v(t) = 1 + 0.5\sin 35t$ is much higher than others due to the high variation rate, as analyzed in Section II. In the remainder

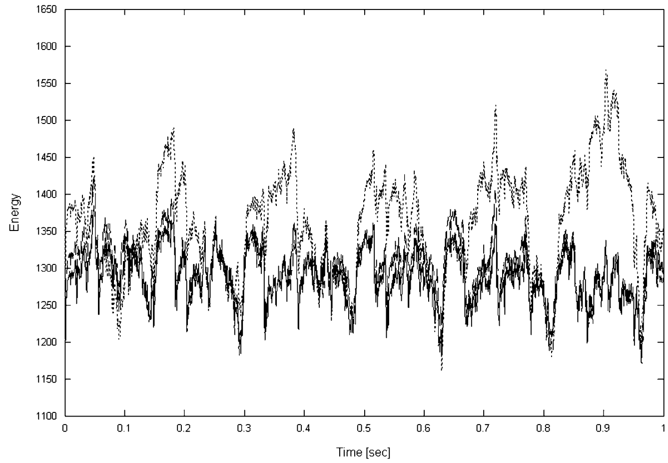

Fig. 2. Energies of uncontrolled translating beams: $v = 1$ (solid), $v(t) = 1 + 0.5 \sin 10t$ (dashed), and $v(t) = 1 + 0.5 \sin 35t$ (dotted).

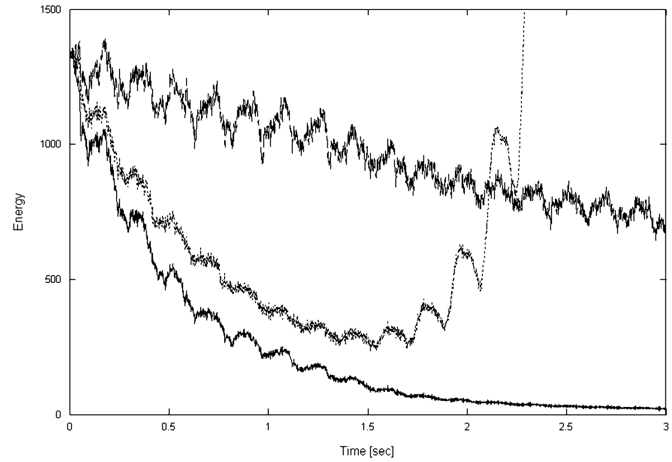

Fig. 4. Energies of open-loop and closed-loop controlled systems with $v(t) = 1 + 0.5 \sin 35t : d_c = 5$ (dotted), $d_c = 50$ (dashed), and $\beta = 0.01$ (solid).

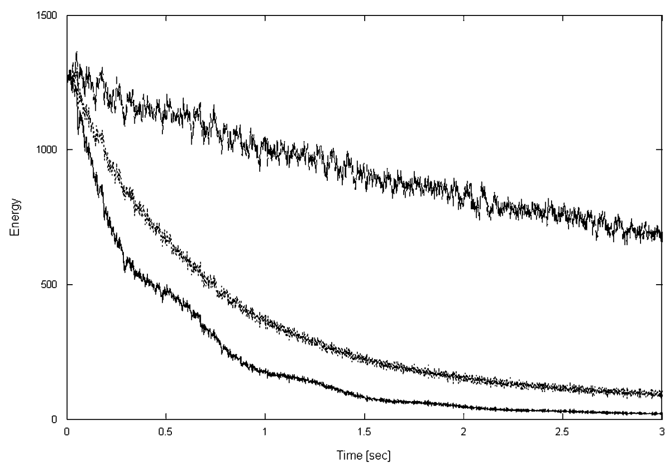

Fig. 3. Energies of open-loop and closed-loop controlled systems with $v(t) = 1 + 0.5 \sin 10t : d_c = 5$ (dotted), $d_c = 50$ (dashed), and $\beta = 0.01$ (solid).

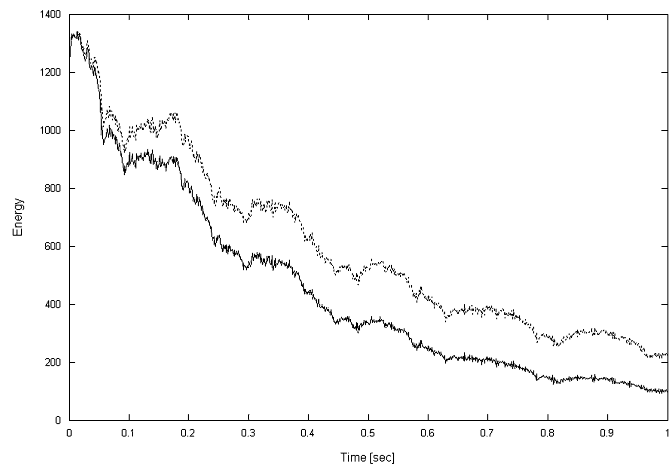

Fig. 5. Energies of closed-loop systems with $v(t) = 1 + 0.5 \sin 35t : \beta = 10^{-4}$ (dotted) and $\beta = 0.5$ (solid).

of this section, the boundary controller proposed will be exerted to the beam systems with $v(t) = 1 + 0.5 \sin 10t$ and $v(t) = 1 + 0.5 \sin 35t$.

B. Comparison of Open/Closed-Loop Controlled Systems

Fig. 3 shows the simulation results of the beam system with $v(t) = 1 + 0.5 \sin 10t$ to compare the open-loop controlled system operated by only dampers setting as $d_c = 5$ (dotted line) and $d_c = 50$ (dashed line) with the closed-loop system having the control gain $\beta = 0.01$ in (13) (solid line), respectively. As shown in Fig. 3, even though all of the three controllers lead to the energy reduction of the time-varying beam, the vibrations of the closed-loop system are more significantly reduced. Further, it is also noted from Fig. 3 that the open-loop controlled system with $d_c = 50$ is going down more slowly than that under $d_c = 5$, despite the higher damping value. This explains that the boundary damper itself cannot effectively suppress the vibration energy of translating continua systems as mentioned in Section II.

For observations of the case with faster varying speed, $v(t) = 1 + 0.5 \sin 35t$ has been also considered with the same controlled conditions as those used in Fig. 3, and the results are presented in Fig. 4. As shown in Fig. 4, even though local increases in the energies of the closed-loop and open-loop controlled systems are taking place due to the fast time-varying properties of the system parameters, the energies are still stabilized without any divergences, except the open-loop controlled system with $d_c = 5$. If considering a system with much faster variation than $v(t) = 1 + 0.5 \sin 35t$, the vibration energy of the beam system surely diverges despite the boundary controllers. However, in actual situations, the worst phenomenon might be unreasonable. The important point to be noted from Fig. 4 is the robustness property of the proposed control law against the uncertainly varying patterns of the moving speed.

C. Effect of Control Gain β in a Closed-Loop System

Fig. 5 describes the vibration energy of the translating beam with $v(t) = 1 + 0.5 \sin 35t$ to show the effectiveness of the control gain β in (13) by comparing the two types of closed-loop system specified by $\beta = 10^{-4}$ (dotted line) and $\beta = 0.5$ (solid line), respectively. Analyzed in Theorem 1, it is seen from Fig. 5 that the vibration energy of the beam controlled by higher β decays more quickly since the exponential index λ in (14) depends on the value of β. However, the value of β might not be set too large due to the limit presented in Theorem 1; here, the limit value is $\beta = 0.5$ following the theorem. Nevertheless, from the simulation results in Fig. 5, it has been observed that the vibrations of the translating tensioned continua can be more effectively suppressed

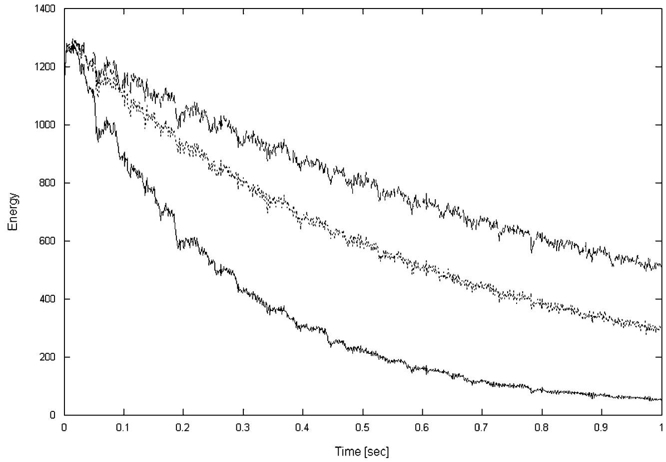

Fig. 6. Energies of open-loop and closed-loop controlled stationary systems with $v = 0 : d_c = 100$ (dotted), $\beta = 0.5$ (dashed), and $\beta = 2$ (solid).

by choosing a properly higher control gain β. From this investigation, we can get a useful clue to controlling translating continua with a very slow moving speed or a stationary continua system.

D. Application to Stationary Continua Systems (i.e., $v = 0$)

Fig. 6 presents the mechanical energies of the stationary beam with $v = 0$ to show the effectiveness of the control gain β, in which the open-loop controlled system having $d_c = 100$ (dotted line) and the two types of closed-loop controlled system specified by $\beta = 0.5$ (dashed line) and $\beta = 2$ (solid line), respectively, are compared. As identified in Section II, the vibration is more quickly suppressed by choosing a higher damper in the case of stationary continua regardless of whether it is tensioned. From Fig. 6, it is noted that, in the case of stationary continua, the performance of the open-loop controlled system is better than that of the closed-loop system with $\beta = 0.5$. However, it is also noted that the better performance is no longer valid when selecting a higher value of the control gain β. That is, the vibration energy of the closed-loop system with $\beta = 2$ converges more quickly when stable than that of the open-loop controlled system. Further, the value of control gain $\beta = 2$ never imposes a heavy burden on the controller to be implementable when comparing with $d_c = 100$. The reason for the stability performance for stationary continua systems can also be easily understood through the solution of the time derivation of $V(t)$ in (12), although further investigation is needed.

From the simulation results and discussions in Sections IV-A–D, it is finally summarized that, under the boundary control law proposed in (13), the vibration energy of translating (or stationary) continua systems with an arbitrarily varying speed can be stabilized and effectively dissipated by setting an appropriate control gain.

V. Conclusion

In this paper, a boundary control scheme for axially moving continua with an arbitrarily varying speed has been proposed. For axially translating continua, two things are essential to design an effective vibration controller: Due to the continuity property of the materials, the slope term at the right boundary should be included in the Lyapunov energy functional. Hence, the slope rate at the right boundary is required as an input signal to the controller. By properly handling this signal, the vibrations of translating continua and stationary as well can be more effectively suppressed.

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Electrostatic Linear Actuator With a Long Stroke Rolling Spring Guide

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***Abstract*—In contrast with diverse design concepts of actuators, we have developed an electrostatic linear actuator integrated with a long stroke rolling spring guide. The rolling spring guide realizes guiding function through rolling movements of two parallel preloaded belt-shaped springs. The electrostatic actuating force is generated by applying electrical fields to the structure of spring guide. Besides the driving voltage, the geometric size and the preloaded span of the spring guide are the main influential parameters of electrostatic actuating force and actuating displacement. With adequate adjustment of the preloaded span, this electrostatic actuator can generate not only a large actuating displacement in μm range, but also a fine positioning displacement in μm range. The finite element analysis (FEA) and the geometric analysis are applied to analyze spring stress and to derive the shape equation of the spring guide. Furthermore, a theoretical model for our electrostatic actuating principle is deduced on the basis of the shape equation. In addition to the theoretical analyses, the performance of the electrostatic actuator is experimentally tested and studied.**

***Index Terms*—Actuators, electrostatic devices, system analysis and design.**

I. Introduction

An electrostatic actuator has many advantages over an electromagnetic actuator. First, as analyzed by Belouschek *et al.* [1], electrostatic force is less affected by volume variation of a system than electromagnetic force. Moreover, low resistance and compact structure enable an electrostatic actuator to be more adaptable to various applications in precision systems. Electrostatic sensors apply geometric variations of opposite electrodes to measure acceleration,

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